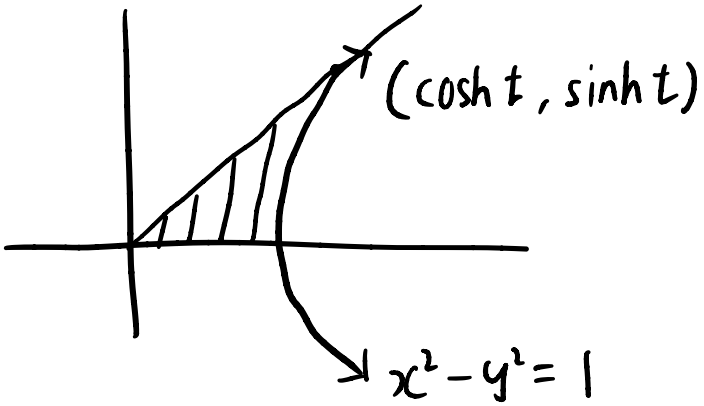
Compute the area of the region below:



Compute the area of the region below:

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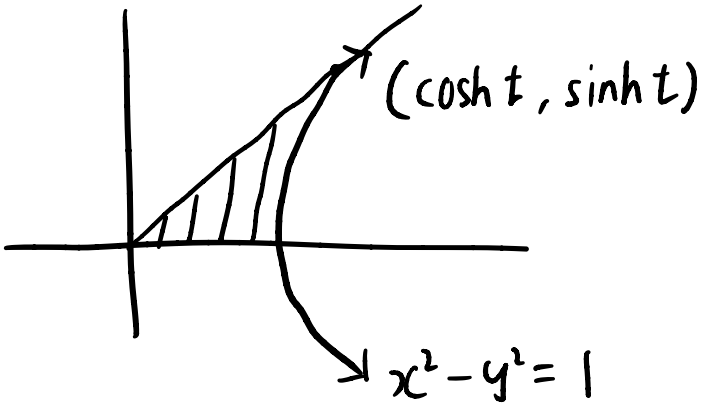

$\sinh^2 u + 1 = \cosh^2 u$

$y = \sinh u$

$\Rightarrow dy = \cosh u \, du$

$y = \sinh t \Rightarrow u = t$

$y = 0 \Rightarrow u = 0$

$$\text{Area} = \int_0^{\sinh t} \sqrt{y^2+1} - \frac{\cosh t}{\sinh t} \cdot y \, dy$$

$$= \int_0^t \sqrt{\sinh^2 u + 1} \cosh u \, du - \frac{\cosh t}{\sinh t} \frac{y^2}{2} \Big|_0^{\sinh t}$$

$$= \int_0^t \cosh^2 u \, du - \frac{\cosh t}{2 \sinh t} \left(\sinh^2 t\right)$$

$$= \int_0^t \frac{e^{2u} + 2 + e^{-2u}}{4} du - \frac{\cosh t \sinh t}{2}$$

$$= \left(\frac{e^{2u}}{8} + \frac{u}{2} + \frac{e^{-2u}}{-8} \right) \Big|_0^t - \frac{\cosh t \sinh t}{2}$$

$$= \left(\frac{e^{2t} - e^{-2t}}{8} + \frac{t}{2} \right) - \frac{e^{2t} - e^{-2t}}{8}$$

$$= \frac{t}{2}$$

$$\frac{\cosh t \sinh t}{2} = \frac{(e^t + e^{-t})(e^t - e^{-t})}{8} = \frac{e^{2t} - e^{-2t}}{8}$$